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A Method for Generating Narrowband Test Signals

Brian H. Maranda

Defence R&D Canada

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Abstract

One common method of testing a narrowband signal detector is to examine its performance when processing a simulated time series consisting of tones in additive noise, the amplitude of the tones being adjustable. The tones are usually implemented as pure sinusoids, but these do not adequately exercise the different analysis bandwidths available in the narrowband detector of a modern passive sonar. A method is described herein for generating a narrowband test signal with controllable bandwidth, achieved by passing white noise through a digital filter. By replicating the filter, a simulated time series can be built to contain multiple tone-like signals of different bandwidths and levels. The digital filter was designed and implemented using a frequency-sampling structure, which allowed a very narrow filter to be realized as a delay memory followed by a small number of second-order filter sections placed in parallel; the operation count is thus small despite the narrow filter width.

Résumé

Une méthode courante pour évaluer un détecteur de signal à bande étroite consiste à examiner sa performance lors du traitement d'une série chronologique simulée composée de tonalités à amplitude réglable et de bruit additif. Les tonalités prennent habituellement la forme de sinusoïdes pures, mais celles-ci n'appliquent pas les stimuli voulus aux différentes bandes passantes d'analyse du détecteur à bande étroite d'un sonar passif moderne. Ce travail présent une méthode pour filtrer de façon numérique du bruit blanc afin de générer un signal d'essai à bande étroite dont la largeur de bande est contrôlable. En reproduisant le filtre, on peut construire une série chronologique simulée contenant de multiples signaux de tonalité de différents niveaux et largeurs de bande. La conception et la mise en œuvre du filtre numérique s'appuient sur une structure d'échantillonnage des fréquences, ce qui a permis de réaliser un filtre très étroit dans lequel une mémoire à retard est suivie d'un petit nombre de sections de filtre de second ordre placées en parallèle; le nombre d'opérations est donc faible malgré l'étroitesse du filtre.

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Executive summary

Background

The impetus for this work was the need to test the narrowband passive detector in IMPACT, the airborne sonar testbed built and maintained by DRDC Atlantic. The results of its operation on experimental data did not appear entirely correct, but without a strictly controlled data set to act as ground truth, there was no way to reach a definite conclusion about its correctness. To provide a proper evaluation of the narrowband processing, a simulated data set with the following properties was desired: (1) the test signals should be narrowband signals of controllable bandwidth, and not pure sinusoids, and (2) the signal-to-noise ratio should be adjustable, to allow testing for the minimum detectable signal.

Results

A method was devised for generating a narrowband test signal with controllable bandwidth, achieved by passing white noise through a narrowband digital filter; a frequency-sampling structure was identified as a suitable means for realizing a very narrow filter. By replicating the filter, a simulated time series could be built to contain multiple tone-like signals of different bandwidths and levels. Software based on this technique was used to simulate a time series and store it on a magnetic tape that could be plugged directly into IMPACT.

Significance of the Work

The short-term significance of this work is that it provided a means for identifying errors in the narrowband passive processing in IMPACT, and for monitoring changes in system performance while the software was being upgraded to eliminate errors and to improve detection performance. The long-term significance is in the methodology for generating narrowband test signals of controllable bandwidth, since pure sinusoids are inadequate when testing a narrowband passive detector that provides different frequency resolutions to the sonar operator. Although the simulation method has been implemented on a general-purpose computer, it would also be a good candidate for real-time embedded applications; e.g., for generating narrowband test signals on a DSP chip or microprocessor as part of a dedicated test system.

Future Work

The simulation software is currently oriented toward generating test data for the IMPACT system, whereas other sonar processors at DRDC Atlantic could benefit from having similar test data available. For this reason, the simulation

method should be integrated into other software test packages as the opportunity arises.

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Sommaire

Contexte

Ces travaux ont été entrepris en réponse au besoin d'évaluer le détecteur passif à bande étroite du système IMPACT, le banc d'essai de sonar aéroporté créé et entretenu par le RDDC Atlantique. Les résultats d'application de ce détecteur à des données expérimentales ne semblaient pas entièrement exacts, mais en l'absence d'un ensemble de données strictement contrôlées servant de données témoins de terrain, aucune conclusion définitive quant à leur exactitude n'était possible. Pour évaluer comme il se doit le traitement à bande étroite, il fallait disposer d'un ensemble de données simulées ayant les propriétés suivantes: (1) les signaux d'essai devaient être des signaux à bande étroite de largeur contrôlable, et non de sinusoïdes pures, et (2) le rapport signal/bruit devait être réglable afin d'évaluer le signal minimal détectable.

Résultats

Une méthode conçue pour générer un signal d'essai à bande étroite dont la largeur de bande est contrôlable consiste à faire passer du bruit blanc dans un filtre numérique à bande étroite; on a identifié une structure d'échantillonnage de fréquences qui permet de réaliser un filtre très étroit. En reproduisant le filtre, on peut construire une série chronologique simulée contenant de multiples signaux de tonalité de différents niveaux et largeurs de bande. Un logiciel basé sur cette technique a été utilisé pour simuler une série chronologique et enregistrer cette série sur un ruban magnétique directement utilisable par le système IMPACT.

Importance des travaux

À court terme, ces travaux ont fourni un moyen d'identifier les erreurs dans le traitement passif en bande étroite du système IMPACT, et de contrôler les variations de la performance du système pendant la mise à niveau du logiciel, qui visait à éliminer les erreurs et à améliorer la performance de détection. À long terme, ils offrent une méthode pour générer des signaux d'essai à bande étroite dont la largeur de bande est contrôlable, car l'utilisation de sinusoïdes pures est inadéquate pour évaluer un détecteur passif à bande étroite offrant différentes résolutions de fréquence à l'opérateur de sonar. Bien que la méthode de simulation ait été mise en œuvre sur un ordinateur universel, elle conviendrait bien aussi pour des applications intégrées fonctionnant en temps réel, p. ex. la génération de signaux à bande étroite sur une puce DSP ou un microprocesseur faisant partie d'un système d'essai spécialisé.

Travaux futurs

Le logiciel de simulation est actuellement orienté vers la production de données d'essai pour le système IMPACT, mais d'autres processeurs de sonar du RDDC Atlantique pourraient bénéficier de données d'essai du même genre. Pour cette raison, la méthode de simulation devrait être intégrée à d'autres progiciels d'essai lorsque l'occasion se présentera.

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1. Introduction

1.1 Background

One of the difficulties in building and maintaining a system that performs a complicated signal-processing task is to verify the correctness of its operation. This is especially true for sonar systems, as many errors that arise in practice lead not to catastrophic system failure, but to a degradation in detection performance that is not obvious to the sonar operators. Even those errors that produce artifacts in the sonar displays may go unnoticed for some time, if it happens that the artifacts resemble signals occurring in practice.

Of the many different ways to test a sonar system for correct operation, one of the most common is to examine its performance when processing a known data set. Often — far too often, in fact — such testing reduces to processing previously recorded experimental data and seeing if the sonar output "looks correct." This procedure is rarely adequate for arriving at a definite conclusion, since there is usually so much uncertainty in the knowledge of the experimental data set that it is impossible to say whether the sonar output is correct or not. To carry out a careful troubleshooting of a sonar system requires strictly controlled input signals, such as can only be generated through simulation.

A case in point is IMPACT, the airborne testbed sonar built and maintained at DRDC Atlantic. Many hours of experimental data had been analyzed on this system over the years, and the passive narrowband detector was thought to be operating correctly, until a comparison of the processor's output with that of another processor revealed deficiencies. This discovery was a spur to re-visiting the design of the narrowband processing and making improvements [1]. It was also a spur to simulating appropriate test data. A simulated time series with the following properties was desired: (1) the test signals should be narrowband signals of controllable bandwidth, and not pure sinusoids, and (2) the signal-to-noise ratio (SNR) should be adjustable, to allow testing for the minimum detectable signal.

Requirement (1) was motivated by the belief that pure sinusoids are inadequate when testing a narrowband passive detector that offers different frequency resolutions to the sonar operator. Pure sinusoids have no frequency extent (their spectra being represented by delta functions), and hence do not exercise the different processing options.

Requirement (2) allows quantitative testing to be done. For example, if theory suggests that the detection threshold is -10 dB, the test set might consist of 21 signals with SNRs ranging from -20 to 0 dB; the detection performance that is actually realized by the sonar is gauged by the SNR of the weakest test signal that is detectable (the "minimum detectable signal", or MDS). It must be kept in mind, however, that the detection of a signal in noise is intrinsically

probabilistic and should be measured statistically with a large experimental sample; nevertheless, although MDS testing provides only a rough performance estimate, it is sufficiently accurate to reveal whether the sonar falls substantially short of its predicted performance.

1.2 Technical Approach

In this paper, a method is devised for producing a simulated time series that meets the above two requirements. The basis of the method is shown in the block diagram of Fig. 1: a signal component is produced by filtering the output of a Gaussian random number generator (RNG), and this signal is added to background noise also produced with a Gaussian RNG. The output samples from each RNG are statistically independent; i.e., each RNG outputs white Gaussian noise. The spectral width and shape of the signal is thus determined completely by the filter, which is assumed to be narrowband. The resulting time series, consisting of a narrowband Gaussian signal in white Gaussian noise, is among those treated by Walker [2] in his performance analysis of narrowband detectors implemented with the fast Fourier transform (FFT).

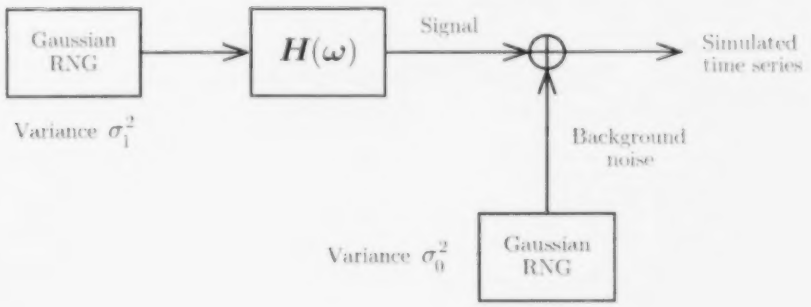

Figure 1: *Method for generating the simulated time series.*

The main technical problem to be faced is the design and implementation of the filter that shapes the signal spectrum, a problem that is complicated by the very narrow bandwidth that is desired (on the order of a binwidth of the sonar processor). There are many possible approaches to the problem, and, after weighing the options, it was decided to employ a frequency-sampling structure. The main advantage of this approach is the simplicity of the software implementation; however, filters explicitly implemented in a frequency-sampling structure are not commonly used in digital signal processing (DSP), and hence it was felt that the method should be documented.

The material that follows is concerned with the generation of a single test signal in noise, as shown in Fig. 1. It is an obvious extension to insert multiple signals into the same simulated time series by replicating the filter. Moreover, by changing a few parameters from filter to filter (while still retaining the same filter structure), a simulated time series can be built to contain multiple tone-like signals of different bandwidths and SNRs.

2. Frequency-Sampling Filters

2.1 Digital Filters

Since the simulated time series is generated on a digital computer, the computational methods used in its generation are based on DSP techniques. Some introductory material on digital filters is now presented, more to establish the notation rather than to explain the textbook fundamentals [3, 4].

The kind of digital filter considered here has a finite-duration impulse response (FIR) of length N, and can thus can specified by a finite set of coefficients $h[n]$, here chosen to be non-zero for $0 \leq n \leq N-1$. The filter can also be characterized by its frequency response $H(\omega)$, which is computed from the filter coefficients via the equation

$$H(\omega) = \sum_{n=0}^{N-1} h[n] e^{-i\omega n}. \tag{1}$$

This form of the equation assumes that the sampling frequency is 1 Hz. It is easily seen that $H(\omega)$ has period 2π, so that we can restrict our attention to the range $-\pi \leq \omega \leq \pi$. For our later work we shall also require the z-transform of $h[n]$, defined by

$$H_z(z) = \sum_{n=0}^{N-1} h[n] z^{-n}. \tag{2}$$

We have the obvious relationship $H(\omega) = H_z(e^{i\omega})$.

Of special significance are the N samples of the frequency response at the frequencies $\omega_k \equiv 2\pi k/N$, these samples being denoted by $H[k] = H(\omega_k)$ for $0 \leq k \leq N-1$. The filter coefficients $h[n]$ and the frequency samples $H[k]$ are connected by the discrete Fourier transform, with forward and inverse relationships

$$\begin{aligned} H[k] &= \sum_{n=0}^{N-1} h[n] W_N^{kn} \\ h[n] &= \frac{1}{N} \sum_{k=0}^{N-1} H[k] W_N^{-kn}, \end{aligned} \tag{3}$$

where $W_N \equiv \exp(-i2\pi/N)$. The quantities $h[n]$ and and $H[k]$ are in some sense equivalent, since either set can be generated from the other and either set completely specifies the filter.

In designing a filter to perform a particular task, we choose the coefficients $h[n]$ so as to obtain a desired frequency response $H(\omega)$; in addition, the coefficients $h[n]$ are constrained to have certain properties that ensure the filter is of a desired class or type. One restriction is that the coefficients $h[n]$ should be real-valued; this condition implies that

$$H^*(\omega) = H(-\omega),$$

where the asterisk denotes complex conjugation. If we then represent $H(\omega)$ in the form

$$H(\omega) = |H(\omega)|e^{i\theta(\omega)}, \tag{4}$$

it follows that

$$|H(-\omega)| = |H(\omega)| \qquad \text{and} \qquad \theta(-\omega) = -\theta(\omega).$$

Another constraint almost invariably imposed is that the filter must have linear phase. The coefficients $h[n]$ must then obey specific symmetry relations, the case of interest here being described by the relation $h[n] = h[N-n-1]$. It may then be shown that

$$H(\omega) = A(\omega)e^{-i\omega(N-1)/2} \tag{5}$$

for a certain function $A(\omega)$. It is important when interpreting Eq. (5) to note that, in general, $A(\omega)$ and $e^{-i\omega(N-1)/2}$ cannot be identified with $|H(\omega)|$ and $e^{i\theta(\omega)}$ in Eq. (4). As explained by Porat [5], the factor $A(\omega)$ is given by $A(\omega) = \pm|H(\omega)|$ with the sign chosen such that the phase can be written in the continuous form $-\omega(N-1)/2$. That is, the notation accommodates phase jumps of $\pm\pi$ by allowing the amplitude to change sign; in the representation of Eq. (4), the phase $\theta(\omega)$ could explicitly contain discontinuous jumps of $\pm\pi$.

As an example, consider a filter with coefficients $h[n] = 1/N$ for $0 \le n \le N-1$; note that the coefficients satisfy the symmetry condition $h[n] = h[N-n-1]$. A simple calculation shows that Eq. (5) holds with

$$A(\omega) = \frac{\sin(\omega N/2)}{N\sin(\omega/2)}, \tag{6}$$

and $A(\omega)$ changes sign as it passes through the zeros of $\sin(\omega N/2)$. In general we expect to see this behavior when the z-transform $H_z(z)$ has zeros on the unit circle.

2.2 Frequency-Sampling FIR Filters

A frequency-sampling FIR filter [6] is little more than an FIR filter for which the design and implementation are carried out in terms of the frequency samples $H[k]$ instead of the time-domain coefficients $h[n]$. As the first step in its derivation, the z-transform of the filter impulse response is expressed in terms of the samples $H[k]$: substituting Eq. (3) into Eq. (2), we find after some manipulation that [6]

$$H_z(z) = \frac{(1 - z^{-N})}{N} \sum_{k=0}^{N-1} \frac{H[k]}{1 - e^{i\omega_k} z^{-1}}, \tag{7}$$

where $\omega_k = 2\pi k/N$ as above. This equation expresses the filter as a cascade of an FIR filter $1 - z^{-N}$ and a parallel connection of filter blocks each with an infinite-duration impulse response (IIR). A block diagram of the corresponding frequency-sampling structure is shown in Fig. 2, where the IIR filter blocks are denoted by $G_k(z)$. When the filter is such that only a small number of samples $H[k]$ are non-zero, only a few IIR blocks are present and this structure becomes a particularly efficient way to implement the filter.

The goal of filter design is to shape the frequency response $H(\omega)$, and hence we must know how $H(\omega)$ depends on the choice of frequency samples $H[k]$. Since $H(\omega) = H_z(e^{i\omega})$, it follows from Eq. (7) that [4, p. 107]

$$H(\omega) = e^{-i\omega(N-1)/2} \sum_{k=0}^{N-1} \frac{H[k] e^{-i\pi k/N} \sin(\omega N/2)}{N \sin\left((\omega - \omega_k)/2\right)}.$$

This equation can be viewed as an interpolation formula that shows how the frequency response $H(\omega)$ at an arbitrary frequency ω can be expressed as a linear combination of the N frequency samples $H[k]$. The application of simple trigonometric relations produces a more convenient form of the interpolation formula [3, p. 98],

$$H(\omega) = e^{-i\omega(N-1)/2} \sum_{k=0}^{N-1} H[k] e^{i\omega_k (N-1)/2} F_N(\omega - \omega_k), \tag{8}$$

where

$$F_N(\omega) \equiv \frac{\sin(N\omega/2)}{N \sin(\omega/2)}. \tag{9}$$

The interpolating function $F_N(\cdot)$ often crops up in analysis of DSP algorithms; more information on its properties can be found in Annex A. Since $F_N(0) = 1$ and $F(\omega_k) = 0$ for $0 < k < N$, it is evident that the interpolation formula correctly yields $H(\omega_k) = H[k]$ at the sampling points $\omega = \omega_k$. The function is plotted in Fig. 3 for $N = 32$; it has a large peak at the origin, and exhibits ripples of decreasing amplitude to the left and right.

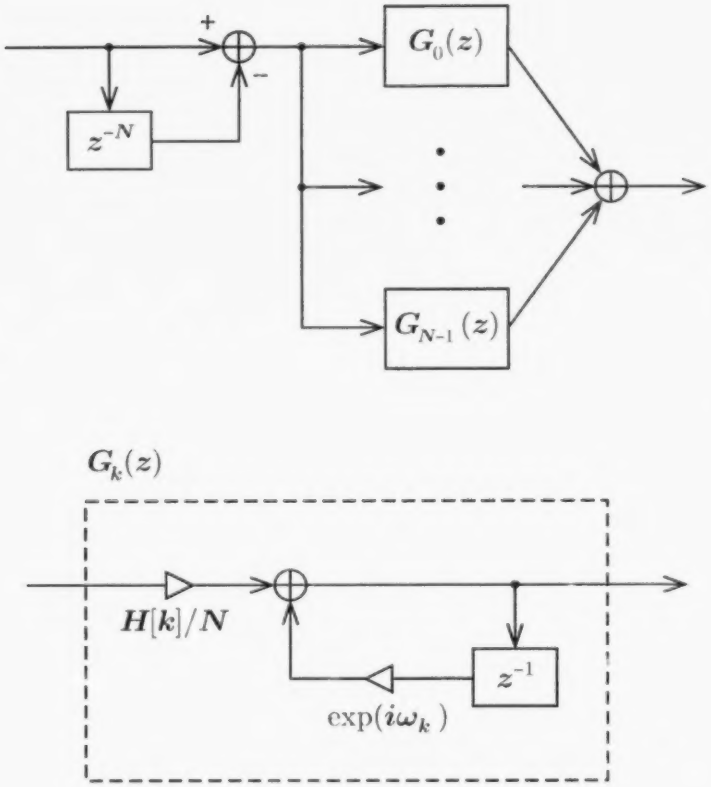

Figure 2: *Implementation of an FIR filter using a frequency-sampling structure.*

The formula of Eq. (8) is entirely general, holding for any FIR filter. If it is now assumed that the filter is a linear-phase filter of the form in Eq. (5), then

$$H(\omega) = e^{-i\omega(N-1)/2} \sum_{k=0}^{N-1} A[k] F_N(\omega - \omega_k), \tag{10}$$

where $A[k] \equiv A(\omega_k)$. Comparison of Eq. (5) and Eq. (10) shows that in fact

$$A(\omega) = \sum_{k=0}^{N-1} A[k] F_N(\omega - \omega_k). \tag{11}$$

This equation shows that the amplitude response $A(\omega)$ is synthesized by summing weighted and shifted versions of the interpolating function $F_N(\cdot)$.

Although N frequency samples $A[k]$ appear in Eq. (11), they are not all distinct owing to symmetry relationships. The cases when N is even or odd are slightly

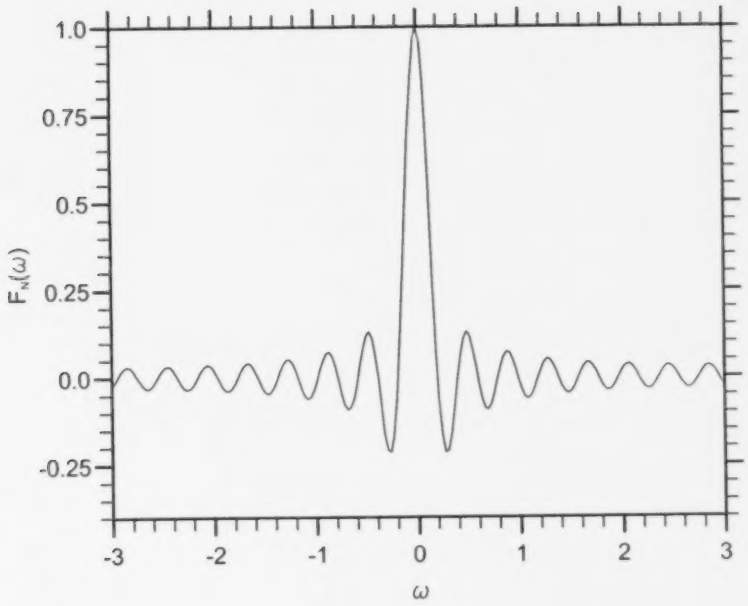

Figure 3: *The interpolation function for* $N = 32$.

different, and in what follows it will be assumed that N is even; justification for this choice will be given later. When N is even, it can be shown that $A[N/2] = 0$ and $A[k] = -A[N-k]$; there are thus a total of $N/2$ independent values for $A[k]$, with indices in the range $0 \leq k \leq N/2 - 1$. It is shown in Annex B that, in terms of the reduced set of coefficients, Eq. (11) becomes

$$A(\omega) = A[0]F_N(\omega) + \sum_{k=1}^{N/2-1} A[k]\{F_N(\omega - \omega_k) + F_N(\omega + \omega_k)\}. \tag{12}$$

This equation was used to compute the filter amplitude response for the numerical examples that appear below.

3. Narrowband Filter Design

The filter-design problem reduces to finding numerical values for the frequency samples $A[k]$ that produce a desired shape for the amplitude response (and hence, in this case, for the spectrum of the test signal). In the following subsections we give a few specific examples of narrowband filters that fit conveniently into the frequency-sampling framework. Since different researchers may have different requirements for an appropriate test signal, there is no single filter that can be regarded as optimal for all situations.

The equations derived earlier will be used as the basis for the design, but one point must first be clarified: these equations assume a 1-Hz sampling frequency (the usual convention in DSP analysis), whereas the test signal is simulated with a sampling frequency f_s equal to the internal sampling frequency of the sonar system under test. The actual frequency of the k-th frequency sample $A[k]$ is $k\,\Delta f$, where $\Delta f \equiv f_s/N$ is the spacing between samples; the temporal sampling period of the digital signals is given by $\Delta t = 1/f_s$, with units of seconds.

In designing the filter, the only parameters under our control are the filter length N and the frequency samples $A[k]$. The frequency spacing $\Delta f = f_s/N$ is adjusted by changing N; usually N will be chosen to make Δf the same as the binwidth of the narrowband detector. In fact, unless otherwise stated, if the narrowband detector uses an FFT of length M, it will be assumed that $N = M$. (Since FFT lengths are normally powers of two, N will be even.) This choice for N locates the frequency samples $A[k]$ of the filter exactly on the bin centers of the FFT, whence it follows that shaping the narrowband filter is much the same as shaping an FFT bin.

3.1 Rectangular Weighting

If we want to simulate a narrowband signal in bin k_0 of the FFT detector (corresponding to true frequency $f_s k_0/N$), we must construct a narrowband filter with its passband centered at k_0. To make the filter as narrow as possible, the first inclination is to let only a single frequency sample be non-zero:

$$A[k] = \begin{cases} 1 & \text{for } k = k_0, \\ 0 & \text{elsewhere.} \end{cases}$$

Figure 4 shows the corresponding amplitude response $|H(\omega)| = |A(\omega)|$ in the vicinity of the passband for the values $k_0 = 25$ and $N = 256$. The frequency axis is labelled in terms of the FFT bin numbers; with the frequency scale defined in this way, the shape of the plotted amplitude response is almost independent of k_0 and N (with the assumption that $N = M$ as discussed above).

It can be shown that the filter impulse response $h[n]$ is a sinusoid at frequency ω_{k_0} with a rectangularly-weighted envelope:

$$h[n] = \frac{2}{N} \cos\left[\omega_{k_0}\left(n - \tfrac{N-1}{2}\right)\right].$$

One expects to see high sidelobe levels for rectangular weighting (due to the sharp transition at the pulse edges), and Fig. 4 indeed shows that the sidelobes immediately adjacent to the passband are down by only about 13 dB. Although the passband of this filter is the narrowest possible for a given filter length N, a narrowband test signal with the spectrum of Fig. 4 would be unsuitable for many purposes because of its high sidelobe level.

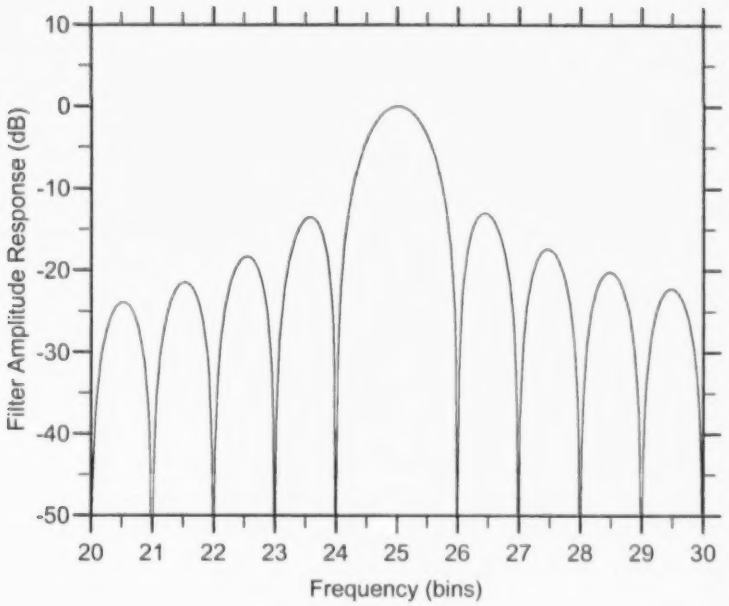

Figure 4: *Narrowband filter response.*

3.2 Hann Weighting

The obvious approach to lowering the sidelobes is to shape the impulse response with a smooth weighting function. However, for an efficient frequency-sampling implementation, the time-domain weighting must lead to a frequency-domain window with only a few non-zero coefficients. One well-known function with this property is the Hann window, which requires only three non-zero coefficients:

$$A[k] = \begin{cases} 1 & \text{for } k = k_0, \\ \frac{1}{2} & \text{for } k = k_0 \pm 1, \\ 0 & \text{elsewhere.} \end{cases} \tag{13}$$

The amplitude response $A(\omega)$ for this filter is shown in Fig. 5. The sidelobes are approximately 31 dB below the mainlobe peak, much lower than those achieved with the rectangular weighting, but the passband is wider. With a Hann weighting the signal will likely be well matched by the narrowband detector (which itself will probably use a Hann or Hamming weighting in performing the spectral analysis).

3.3 Approximating a Rectangular Spectrum

Suppose that we are striving for a signal with a rectangular spectrum (as is sometimes used in theoretical studies [2]), with the proviso that the signal

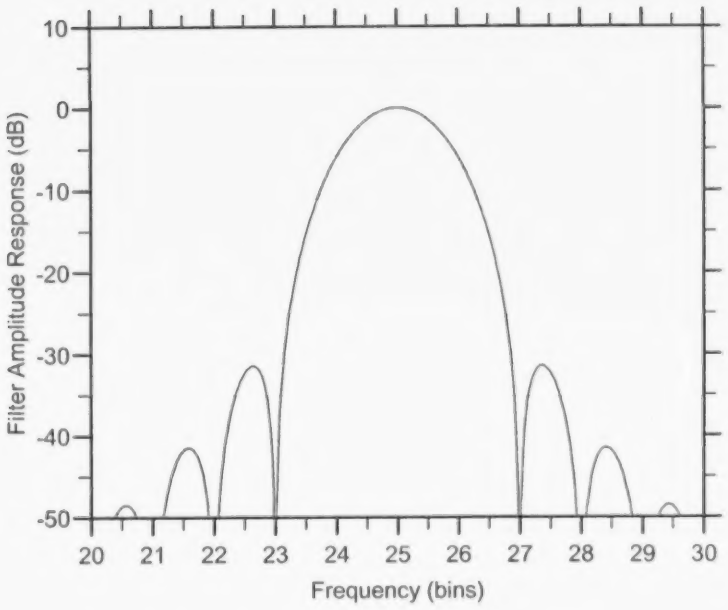

Figure 5: *Narrowband filter response for Hann weighting.*

should remain fairly narrow. To get more flexibility in shaping the filter response, we set $N = 4M$; there are then four frequency samples that can be adjusted over an FFT binwidth. (The center of the k-th bin of the FFT corresponds to frequency sample $A[4k]$.) An intuitive approach to forming a rectangular response is to insert additional 1's in the passband of Eq. (13); for example

$$A[k] = \begin{cases} 1 & \text{for } 4k_0 - 3 \leq k \leq 4k_0 + 3, \\ \frac{1}{2} & \text{for } k = 4k_0 \pm 4, \\ 0 & \text{elsewhere,} \end{cases}$$

where there are 7 samples in the passband and two in the transition band, for a total of 9 non-zero frequency samples.

The amplitude response for this filter is shown in Fig. 6. Despite the *ad hoc* nature of the design, the response approximates a rectangle quite well. By making small adjustments in the values of the frequency samples (particularly the two in the transition band), one could attempt to optimize for the desired rectangular shape. The main disadvantage of this filter is that the number of non-zero frequency samples has been increased by a factor of three from the previous design (3 samples increased to 9), and the computational load is increased by approximately the same amount. Note, however, that the length N of the filter impulse response has been increased by a factor of four.

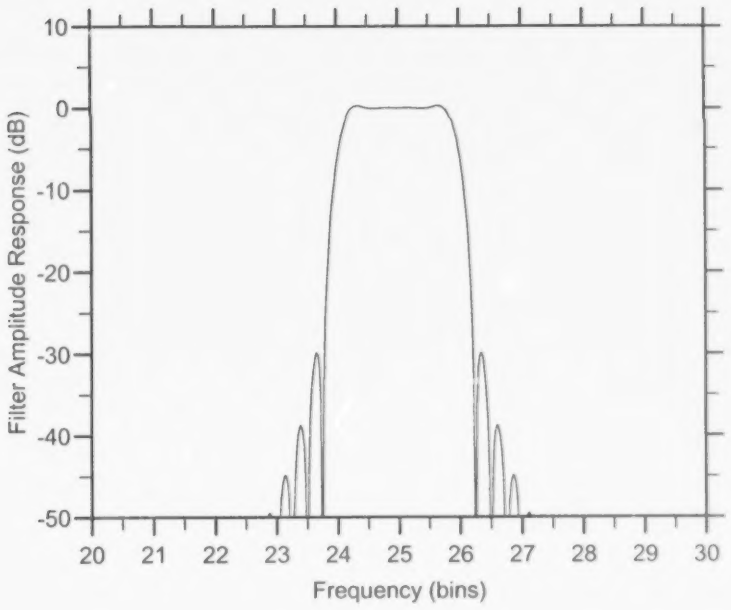

Figure 6: *Narrowband filter with approximate rectangular passband.*

4. Implementation Issues

4.1 Signal-to-Noise Ratio (SNR)

The discussion so far has focussed on simulating the narrowband signal; however, to test detection performance, this signal must be added into a noise background to yield a specified SNR. The main problem in calculating the SNR is to determine how the signal power depends on the level of the noise at the input to the narrowband shaping filter. Explicit formulas are derived for this purpose in Annex C; here we consider how the formulas are used when setting up a simulation.

By definition, the signal-to-noise ratio is

$$\mathrm{SNR} = P_s/P_n,$$

where P_s is the average signal power and P_n the average noise power in a 1-Hz band. It is shown in Annex C that the SNR can be expressed as

$$\mathrm{SNR} = \tfrac{1}{2} f_s \gamma \frac{\sigma_1^2}{\sigma_0^2}, \tag{14}$$

where

$$\gamma \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(\omega)|^2 d\omega$$

is called the *noise gain* of the filter. Here σ_0^2 and σ_1^2 are the variances of the Gaussian RNGs used in generating the simulated time series; see Fig. 1. For computational purposes, we can use the convenient formula

$$\gamma = \frac{1}{N}\sum_{k=0}^{N-1} A^2[k] = \frac{1}{N}\Big\{A[0] + 2\sum_{k=1}^{N/2-1} A^2[k]\Big\},$$

expressing the noise gain directly in terms of the frequency samples.

Now, because we are only interested in achieving a specific signal-to-noise *ratio*, the absolute level of the background noise is not important, and any convenient assignment can be made for σ_0^2. The value of σ_1^2 required to achieve a specified SNR can then be computed from the formula

$$\sigma_1^2 = \frac{2\cdot \mathrm{SNR}}{f_s \gamma}\,\sigma_0^2, \tag{15}$$

obtained by re-arranging Eq. (14). That is, by driving the narrowband filter with a Gaussian random-number generator that outputs samples of variance σ_1^2, a signal of the correct level is produced.

4.2 Software Implementation

One possible realization of a frequency-sampling FIR filter was shown in Fig. 2. A drawback of that realization is its use of complex arithmetic, as it would be more convenient to have a structure in which only real quantities appear. As shown in Annex B, symmetry can be used to reduce Eq. (7) to

$$H_z(z) = \frac{(1-z^{-N})}{N}\left\{\frac{A[0]}{1-z^{-1}} + \sum_{k=1}^{N/2-1} \frac{A[k](-1)^k 2\cos\left(\frac{\pi k}{N}\right)(1-z^{-1})}{1-2z^{-1}\cos\omega_k + z^{-2}}\right\}, \tag{16}$$

in which only real coefficients appear. This equation leads to a structure similar to that shown in Fig. 2, but in which the first-order IIR blocks have been replaced by second-order IIR blocks with real coefficients (and one first-order block with pole $z = 1$).

There is still one practical hurdle to be overcome: Each second-order section

$$\frac{1}{1-2z^{-1}\cos\omega_k + z^{-2}}$$

in Eq. (16) has poles on the unit circle, namely at $z = \exp(\pm i\omega_k)$. Because the filter is implemented using finite-precision arithmetic, round-off error could perturb the poles slightly outside the unit circle, making the filter unstable. The remedy is to implement the filter according to the z-transform $H_z(r^{-1}z)$, where the factor r is slightly less than unity; in this way, the poles are brought

just inside the unit circle, ensuring stability. With r sufficiently close to unity, the change to the filter response is negligible.

With these minor modifications, a frequency-sampling filter was implemented as a subroutine in the C programming language on the basis of Eq. (16); this subroutine formed part of a software application for generating test signals. The code should be portable to any computer that has a full implementation of ANSI C. For testing IMPACT, the simulated signals were generated using the Hann-window design of Eq. (13), for which there are three second-order sections in the filter implementation. The signal bandwidth was controlled over a wide range by changing the filter length N in powers of two, corresponding to the range of FFT lengths used in IMPACT.

The term z^{-N} in Eq. (16), representing a delay by N sample periods, was implemented in the software as a circular buffer of length N words. This is the only term in Eq. (16) that depends on N, and hence filters with different bandwidths differ in their implementation only in the length of their associated circular buffers, the parallel IIR blocks being identical. When N is very large (e.g., 512K), and many filters are implemented in parallel, a significant amount of computer memory may be required. Indeed, the practical upper bound on the number of signals that can be simulated will likely be determined by the capacity of the computer memory.

5. Summary

A method has been described for generating a test signal by passing Gaussian white noise through a narrowband filter. The signal thus has a spectrum whose shape is the same as the filter amplitude response. A narrowband signal of this kind overcomes one deficiency of a pure sinusoid, when used as a test signal: because a sinusoid has no frequency width, it does not adequately exercise the different analysis resolutions implemented in many narrowband sonar detectors.

The technical novelty of the design lies in the use of a frequency-sampling structure to implement the narrowband FIR filter, in contrast to a standard implementation based on time- or frequency-domain convolution. Although the theory of the frequency-sampling filter may appear complicated, the end result is a very narrow filter that can be realized in a simple time-domain structure, consisting of a delay buffer cascaded with a few second-order IIR sections placed in parallel. The bandwidth of the narrowband filter (and hence of the test signal) can be altered merely by changing the length of the delay buffer.

Software to generate simulated time series using this method has been written in the C programming language, one output being a test tape for the IMPACT system that exercises the different frequency resolutions available to the operator. The software is currently resident on a standard Intel-based PC.

Although the simulation method was implemented on a general-purpose computer, it would also be a good candidate for real-time embedded applications; e.g., for generating narrowband test signals on a DSP chip or microprocessor as part of a dedicated test system. In such applications, performing long FFTs for frequency-domain convolution is often undesirable, and time-domain convolution is simply too time-consuming when the filter impulse response is long. The low operation count of a frequency-sampling filter may then prove advantageous, the downside being the requirement for a long delay memory.

Annex A
Interpolating Function

We consider some properties of the function

$$F_N(x) = \frac{\sin(Nx/2)}{N\sin(x/2)}.$$

This function usually arises via the summation formula

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{inx} = e^{ix(N-1)/2}F_N(x),$$

which appears in the analysis of such signal-processing operations as spectral analysis and beamforming. Re-arranging, we have

$$F_N(x) = e^{-ix(N-1)/2}\,\frac{1}{N}\sum_{n=0}^{N-1} e^{inx} = e^{ix(N-1)/2}\,\frac{1}{N}\sum_{n=0}^{N-1} e^{-inx}, \tag{A.1}$$

where the last equality holds because $F_N(x)$ is real-valued and hence is unchanged when its complex conjugate is taken. Hence the function is even: $F_N(-x) = F_N(x)$.

Care has to exercised in evaluating $F_N(x)$ at those points where its denominator is zero; i.e., at $x = 2\pi\ell$ for integral values of ℓ. These points are all removable singularities, and in fact $F_N(2\pi\ell) = (-1)^{(N-1)\ell}$. This last follows by noting that $F_N(0) = 1$ and using trigonometric identities to find

$$\begin{aligned} F_N(x + 2\pi\ell) &= (-1)^{(N-1)\ell}\,F_N(x) \\ &= \begin{cases} F_N(x) & \text{for } N \text{ odd,} \\ (-1)^{\ell}F_N(x) & \text{for } N \text{ even.} \end{cases} \end{aligned}$$

As a specific instance of this result, we have

$$\begin{aligned} F_N(\omega - \omega_k) &= (-1)^{(N-1)}F_N(\omega + \omega_{N-k}) \\ F_N(\omega - \omega_k) &= (-1)^{(N-1)}F_N(\omega - \omega_{N+k}) \end{aligned} \tag{A.2}$$

where $\omega_k = 2\pi k/N$.

Finally, we prove an orthogonality relationship for the integral

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} F_N(\omega - \omega_k)F_N(\omega - \omega_\ell)\,d\omega.$$

Using the formulas in Eq. (A.1), we write

$$\begin{aligned} & F_N(\omega-\omega_k)F_N(\omega-\omega_\ell) \\ & \quad = \frac{1}{N^2}\left[e^{-\imath(\omega-\omega_k)(N-1)/2}\sum_{m=0}^{N-1} e^{\imath m(\omega-\omega_k)}\right]\left[e^{\imath(\omega-\omega_\ell)(N-1)/2}\sum_{n=0}^{N-1} e^{-\imath n(\omega-\omega_\ell)}\right] \\ & \quad = \frac{1}{N^2}e^{\imath(\omega_k-\omega_\ell)(N-1)/2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} e^{\imath\omega(m-n)}e^{-\imath(m\omega_k-n\omega_\ell)}. \end{aligned}$$

But

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{\imath\omega(m-n)}\,d\omega = \delta_{m,n}$$

where $\delta_{m,n}$ is the Kronecker delta, so that

$$\begin{aligned} \frac{1}{2\pi}\int_{-\pi}^{\pi} F_N(\omega-\omega_k)F_N(\omega-\omega_\ell)\,d\omega &= \frac{1}{N^2}e^{\imath(\omega_k-\omega_\ell)(N-1)/2}\sum_{n=0}^{N-1} e^{-\imath n(\omega_k-\omega_\ell)} \\ &= \frac{1}{N}F_N(\omega_k-\omega_\ell). \end{aligned}$$

But $\omega_k - \omega_\ell = \omega_{k-\ell}$, and from the definition of $F_N(\cdot)$ we have the orthogonality formula

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} F_N(\omega-\omega_k)F_N(\omega-\omega_\ell)\,d\omega = \begin{cases} 1/N & \text{for } k=\ell, \\ 0 & \text{otherwise.} \end{cases}$$

Here it is assumed that both k and ℓ are in the range 0 to $N-1$.

Annex B
Exploitation of Symmetry

B.1 Introduction

Although there are N frequency samples $A[k]$, owing to the presence of symmetry they are not all distinct. Different symmetries exist depending on whether N is even or odd; only the case when N is even will be examined. In that case, it can be shown that $A[N/2] = 0$ and $A[k] = -A[N-k]$. The first of these equations states that the frequency sample at the fold-over point (corresponding to actual frequency $f_s/2$) is zero; the second states that the samples must be anti-symmetric about the fold-over point. As is shown below, these facts can be exploited to re-write some of the equations that express the frequency-sampling filter in terms of the frequency samples $A[k]$.

B.2 Interpolation Formula for the Frequency Response

Consider the interpolation formula in Eq. (11), reproduced here as Eq. (B.1):

$$A(\omega) = \sum_{k=0}^{N-1} A[k] F_N(\omega - \omega_k). \tag{B.1}$$

Making the decomposition

$$A(\omega) = A[0]F_N(\omega) + \sum_{k=1}^{N/2-1} A[k]F_N(\omega - \omega_k) + \sum_{k=N/2+1}^{N-1} A[k]F_N(\omega - \omega_k),$$

we can use the symmetry relation and Eq. (A.2) to find

$$\begin{aligned}
\sum_{k=N/2+1}^{N-1} A[k]F_N(\omega - \omega_k) &= \sum_{k=N/2+1}^{N-1} (-A[N-k])(-1)^{(N-1)} F_N(\omega + \omega_{N-k}) \\
&= \sum_{k=N/2+1}^{N-1} A[N-k]F_N(\omega + \omega_{N-k}) \\
&= \sum_{k=1}^{N/2-1} A[k]F_N(\omega + \omega_k).
\end{aligned}$$

Hence we have the equation

$$A(\omega) = A[0]F_N(\omega) + \sum_{k=1}^{N/2-1} A[k]\big\{F_N(\omega - \omega_k) + F_N(\omega + \omega_k)\big\},$$

given as Eq. (12) in the main body of the paper.

B.3 Real Form of the Frequency-Sampling Filter

The first-order IIR blocks $G_k(z)$ of Fig. 2 each implement a single complex pole. However, the poles occur as complex conjugates in the z-plane, and, as is now demonstrated, can be paired together to build second-order sections with real coefficients.

First, for a linear-phase filter with a frequency response as in Eq. (5), the z-transform in Eq. (7) becomes

$$\begin{aligned} H_z(z) &= \frac{(1-z^{-N})}{N} \sum_{k=0}^{N-1} \frac{A[k] e^{-i\omega_k (N-1)/2}}{1 - e^{i\omega_k} z^{-1}} \\ &= \frac{(1-z^{-N})}{N} \sum_{k=0}^{N-1} \frac{A[k](-1)^k e^{i\pi k/N}}{1 - e^{i\omega_k} z^{-1}}. \end{aligned}$$

We now specialize to the case when N is even; we then decompose the summation as

$$H_z(z) = \frac{1-z^{-N}}{N} \left\{ \frac{A[0]}{1-z^{-1}} + \sum_{k=1}^{N/2-1} \frac{A[k](-1)^k e^{i\pi k/N}}{1 - e^{i\omega_k} z^{-1}} + \sum_{k=N/2+1}^{N-1} \frac{A[k](-1)^k e^{i\pi k/N}}{1 - e^{i\omega_k} z^{-1}} \right\}$$

and rewrite the last term as

$$\begin{aligned} \sum_{k=N/2+1}^{N-1} \frac{A[k](-1)^k e^{i\pi k/N}}{1 - e^{i\omega_k} z^{-1}} &= \sum_{k=1}^{N/2-1} \frac{A[N-k](-1)^{(N-k)} e^{i\pi (N-k)/N}}{1 - e^{i2\pi (N-k)/N} z^{-1}} \\ &= \sum_{k=1}^{N/2-1} \frac{A[k](-1)^k e^{-i\pi k/N}}{1 - e^{-i\omega_k} z^{-1}}. \end{aligned}$$

We have once again invoked that fact that N is even to write $(-1)^N = 1$. Then

$$H_z(z) = \frac{(1-z^{-N})}{N} \left\{ \frac{A(0)}{1-z^{-1}} + \sum_{k=1}^{N/2-1} A[k](-1)^k \left[\frac{e^{i\pi k/N}}{1 - e^{i\omega_k} z^{-1}} + \frac{e^{-i\pi k/N}}{1 - e^{-i\omega_k} z^{-1}} \right] \right\},$$

a form clearly exhibiting the conjugate poles at $z = \exp(\pm i\omega_k)$. Combining the two terms in square brackets leads to an expression in which only real-valued coefficients appear [6]:

$$H_z(z) = \frac{(1-z^{-N})}{N} \left\{ \frac{A[0]}{1-z^{-1}} + \sum_{k=1}^{N/2-1} \frac{A[k](-1)^k 2\cos\left(\frac{\pi k}{N}\right)(1-z^{-1})}{1 - 2z^{-1}\cos\omega_k + z^{-2}} \right\}.$$

This equation appears as Eq. (16) in the main body of the paper.

Annex C
Computation of the SNR

The purpose of this Appendix is to derive explicit formulas for the average signal and noise powers P_s and P_n, yielding Eq. (14) for the SNR. The use of power spectral density (PSD) in computing the power of random waveforms will be taken for granted in what follows; a suitable reference is [7]. The simulated time series will be viewed as a sampled version of an analog electrical signal, as would result when a sonar processor digitizes real-world data, so that the units of PSD are V^2/Hz. As suggested by these units, the power calculations must be performed on a true frequency scale (in Hz), and hence the sampling frequency f_s must be introduced into the analysis.

C.1 Signal Power

The first step is to compute P_s, the average signal power at the output of the narrowband filter. The filter's frequency response as a function of frequency f in Hertz is

$$\widetilde{H}(f) = H(\omega), \tag{C.1}$$

where $H(\omega)$ is given in Eq. (1) and $\omega = 2\pi f \Delta t$. (Recall that $\Delta t = 1/f_s$ is the sampling period.) Denoting the one-sided PSD of the noise at the filter input by N_1, the PSD of the filter output is given by

$$S(f) = \tfrac{1}{2} N_1 |\widetilde{H}(f)|^2$$

in the two-sided frequency band $-\frac{1}{2}f_s \le f \le \frac{1}{2}f_s$. The total signal power is then given by the integral

$$P_s = \int_{-\frac{1}{2}f_s}^{\frac{1}{2}f_s} S(f)\, df = \tfrac{1}{2} N_1 \int_{-\frac{1}{2}f_s}^{\frac{1}{2}f_s} |\widetilde{H}(f)|^2 df.$$

Using Eq. (C.1), a change of variable gives

$$\int_{-\frac{1}{2}f_s}^{\frac{1}{2}f_s} |\widetilde{H}(f)|^2 df = f_s \gamma,$$

where

$$\gamma \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(\omega)|^2 d\omega \tag{C.2}$$

is the *noise gain* of the filter [5]. Thus we can write

$$P_s = \tfrac{1}{2} f_s \gamma N_1$$

as our final formula for the signal power.

Although we now have everything we need to compute P_s, the integral in Eq. (C.2) is obviously inconvenient for computational purposes. However, as will now be demonstrated, it is possible to express γ directly in terms of the frequency samples. Since $|H(\omega)|^2 = A^2(\omega)$, we have from Eq. (11) that

$$\gamma = \sum_{k=0}^{N-1} \sum_{\ell=0}^{N-1} A[k]A[\ell] \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} F_N(\omega - \omega_k) F_N(\omega - \omega_\ell)\, d\omega,$$

and it follows from an orthogonality result in Appendix A that

$$\gamma = \frac{1}{N} \sum_{k=0}^{N-1} A^2[k]. \tag{C.3}$$

Since the filter design has been done in terms of the frequency samples $A[k]$ this formula provides a simple way to compute γ; although a relation of the Parseval type yields the alternative expression

$$\gamma = \sum_{n=0}^{N-1} h^2[n],$$

in a frequency-sampling design the coefficients $h[n]$ of the impulse response are often not even computed.

C.2 Background Noise Power

The computation of the background noise power is trivial. If N_0 denotes the one-sided PSD of the white noise, the power in a 1-Hz band is simply

$$P_n = \left(N_0 \ \frac{\mathrm{V}^2}{\mathrm{Hz}}\right)(1 \text{ Hz}) = N_0 \ \mathrm{V}^2.$$

That is, P_n is numerically equal to N_0, but has units of power (V^2) instead of power density (V^2/Hz).

C.3 Signal-to-Noise Ratio

Combining the above equations for P_s and P_n gives the following formula for the signal-to-noise ratio:

$$\mathrm{SNR} = \tfrac{1}{2} f_s \gamma \frac{N_1}{N_0}. \tag{C.4}$$

For sampled white noise, the variance of the digital samples is related to the power spectral density of the noise by

$$\sigma_0^2 = \tfrac{1}{2} f_s N_0 \qquad \text{and} \qquad \sigma_1^2 = \tfrac{1}{2} f_s N_1.$$

Then Eq. (C.4) becomes

$$\mathrm{SNR} = \tfrac{1}{2} f_s \gamma \frac{\sigma_1^2}{\sigma_0^2},$$

which appears in the main body of the paper as Eq. (14).

C.4 Discussion

It should be noted that the signal power as computed by Eq. (C.3) represents the *total* power that passes through the narrowband filter, and consequently includes the power in the sidelobes as well. This situation may be contrasted to that of a pure sinusoid, for which the power is totally concentrated at the signal frequency. One might query whether, for the signal produced by filtering, a more operationally useful definition of signal power should include only the power concentrated in the vicinity of the filter passband and ignore the power distributed throughout the sidelobes. The question is largely moot, since a good test signal will be designed to have low sidelobe levels. In that case, the power entering through the sidelobes will be negligible, and the total power as computed here will be essentially equal to the power entering through the passband.

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